Theory and notation

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1 Abstract

The models behind the analyses and the notation of the help pages.

2 Notation

```
stab_notation(
    format = c("table", "path", "tex")[1]
  )
```

```
                                                 symbol            text     kind
 1:                                                   G               G   scalar
 2:                                                   E               E   scalar
 3:                                                   B               B   scalar
 4:                                              i, j, k           i j k    index
 5:                         \\mathbf{Y} = (\\bar{y}_{ij})               Y   matrix
 6: \\bar{y}_{i\\cdot}, \\bar{y}_{\\cdot j}, \\bar{y}   y_i. y_.j y..   scalar
 7:                                \\mathbf{Z} = (z_{ij})               Z   matrix
 8:                                                 I_j             I_j   scalar
```

```
 9:                                      s^2, f_e          s2 f_e   scalar
10:                                      \\beta_i             b_i   scalar
11:                                    S^2_{d_i}           S2d_i   scalar
12:                                          W_i             W_i   scalar
13:                                   \\sigma^2_i        sigma2_i   scalar
14:                                    \\lambda_n        lambda_n   scalar
15:  \\boldsymbol{\\gamma}_n, \\boldsymbol{\\delta}_n gamma_n delta_n   vector
16:                                            P               P   scalar
17:                \\mathbf{g}_i, \\mathbf{e}_j         g_i e_j   vector
18:                                    \\mathbf{a}               a   vector
19:                      \\alpha, Z_{1-\\alpha}         alpha z   scalar
20:                                          P_i             P_i   scalar
21:                        S^{(1)}_i, S^{(2)}_i       S1_i S2_i   scalar
22:               \\mathbf{A}^\\top, \\mathbf{1}           A^T 1 operator
                                        symbol            text     kind

 1:                                                                    number
 2:                                                                number of
 3:                                                  number of replicates in each
 4:                                                          genotype, environmen
 5:                                                    table of genotype-by-envir
 6:                                                     genotype, environment and
 7:                                          interaction effects (means centred on rows
 8:                                             environment index, environment mean minu
 9:                                  pooled error mean square of the plots and its degree
10:                                          regression slope of genotype i on the envir
11:                                                  Eberhart and Russell's deviat
12:                                                                  Wricke'
13:                                                               Shukla's stabil
14:                              n-th singular value (AMMI: of Z; GGE: of the environment-ce
15:                                                  genotype and environment sing
16: singular value partition of the GGE biplot: 1 genotype-focused, 0.5 symmetric, 0 enviror
17:                                                GGE biplot points of genotype i and e
18:                                                  direction of the average envi
19:    risk of a low yield accepted in the safety-first and reliability indices, and the nor
20:                                                                Lin and Binns' superio
21:                                          Huehn's mean absolute rank difference and varia
22:                                                                transpose, ve

   argument
 1:     .gen
 2:     .env
 3:     .rep
 4:
 5:       .y
 6:
```

```
 7:
 8:
 9:
10:
11:
12:
13:
14:
15:
16:     .svp
17:
18:
19:    alpha
20:
21:
22:
    argument
```

3 The two-way table

All methods start from the table of means $\mathbf{Y} = (\bar{y}_{ij})$, G genotypes by E environments, each the mean of B replicates. The additive model and the interaction:

$$\bar{y}_{ij} = \mu + g_i + e_j + z_{ij}, \qquad z_{ij} = \bar{y}_{ij} - \bar{y}_{i\cdot} - \bar{y}_{\cdot j} + \bar{y}_{\cdot}$$

Regression methods model the interaction as a linear function of the environment index $I_j = \bar{y}_{\cdot j} - \bar{y}_{\cdot}$: $\bar{y}_{ij} = \mu_i + \beta_i I_j + \delta_{ij}$.

Variance methods summarise each row of $\mathbf{Z}$: Wricke's ecovalence $W_i = \sum_j z_{ij}^2$ and Shukla's stability variance $\sigma_i^2 = [G(G-1)W_i - \sum_k W_k]/[(E-1)(G-1)(G-2)]$.

AMMI decomposes $\mathbf{Z}$ by its singular values, $\mathbf{Z} = \sum_n \lambda_n \boldsymbol{\gamma}_n \boldsymbol{\delta}_n^\top$, and tests each term.

GGE decomposes the environment-centred table $\mathbf{X} = (\bar{y}_{ij} - \bar{y}_{\cdot j})$, genotype main effect plus interaction, and plots its first two terms.

4 Scales

The analyses of variance are on the scale of the plots; the stability statistics on the scale of the means. A mean square of the means corresponds to a plot mean square divided by B; the pooled error of the means is s^2/B.

5 Summary

One table of means, four views of it: regression, variance, AMMI and GGE.

6 References

Becker, H. C. and Leon, J. (1988). Stability analysis in plant breeding. *Plant Breeding*, 101(1), 1–23.

Gauch, H. G. (1992). *Statistical Analysis of Regional Yield Trials: AMMI Analysis of Factorial Designs*. Elsevier, Amsterdam.

Yan, W., Hunt, L. A., Sheng, Q. and Szlavnics, Z. (2000). Cultivar evaluation and mega-environment investigation based on the GGE biplot. *Crop Science*, 40(3), 597–605.